Design-Oriented Identification of Critical Times in Transient Response

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Many optimum design problems involve constraints that have to be satisfied for an entire range of certain parameters. For example, in a structure under dynamic loads, the stress constraints have to be satisfied over a given range of time. A parametric constraint may be replaced by equivalent critical point constraints at its local minima for optimization purposes. For many design applications, the evaluation of constraints on transient response is expensive and finding the critical points by calculating the response for many time points can be prohibitively expensive. This is particularly true for optimization applications in which many configurations of the design are examined. The present paper describes three techniques for reducing the computational effort involved in identifying the critical time points. The first approach is an adaptive search technique, well suited for a slowly varying, exactly known response. The second technique, which is useful for noisy response, is based on approximating the response using least-squares splines. The third approach, suited for highly oscillatory response, is based on grouping closely spaced local peaks to identify a single superpeak. The error incurred due to superpeak switching is compared with the errors due to commonly employed constraint approximations. Two example problems are considered to demonstrate the computational efficiencies of the proposed techniques.

Introduction

MANY optimum structural design problems involve constraints that have to be satisfied for an entire range of certain parameters. For example, when a structure is designed subject to dynamic loads, the stresses induced will vary with time and stress constraints that have to be satisfied over a given range of time. One method of dealing with this type of time-varying constraint is to replace it by an integrated constraint (e.g., see Refs. 1-6). The drawback of this integral approach is that it tends to delay the warning of an impending constraint violation over a small time interval. A second method of handling a time-dependent constraint is to replace it by a finite number of constraints placed at closely spaced discrete times (e.g., see Ref. 7). This approach increases the number of constraints and thereby the cost of the constrained optimization problem.

An attractive alternative to these two approaches is to monitor the constraints only at the critical points (constraint peaks) and to update these points as the optimization progresses (e.g., see Refs. 8 and 9). The efficiency (compared to the second method) of the critical point approach is due to the reduced number of constraints and the minimal computational effort required for calculating the derivatives of the critical point constraints with respect to the design variables during the optimization process.

The accurate calculation of critical time points requires that the analysis be repeated at closely spaced time points. For many engineering design applications, the evaluation of behavior constraints such as stresses requires a costly finite element analysis and the calculation of these constraints for many time points can be very expensive. The cost consideration is particularly important in optimization applications because many configurations of the structure are examined during the optimization process. This paper describes three techniques for reducing the computational effort involved in identifying the critical time points. In the discussion of the three methods, it is assumed that the transient response may be sampled at a large number of discrete points. The techniques developed below seek to minimize the number of evaluations required to identify the critical points.

The first approach is an adaptive search technique, similar to a one-dimensional optimization method. This technique employs the magnitude of the constraint value at a particular time point to determine the step increment for the next constraint evaluation. Since the constraint response functions can be multimodal, the search is continued throughout the given range of time to find all of the critical points and the corresponding constraint values. The precise location of the critical time point is obtained via a quadratic approximation based on three or five data points. The adaptive search technique is particularly suited for a slowly varying, exactly known response where the number of critical points is small. The second approach is based on approximating the response of the structure using splines. Splines provide a smoother approximation to constraint functions than any other approximating functions. In this study, least squares parabolic splines are used because of their efficiency in representing noisy response histories.

The third technique is based on grouping several closely spaced local peaks to identify a single superpeak. This technique is useful for a fast varying response, where there are many critical points at closely spaced time intervals. The errors due to monitoring constraints at superpeaks instead of at all local peaks are compared with errors incurred due to commonly employed constraint approximations. An analytical example and a 25 bar space truss structure subjected to earthquake loading are considered for demonstrating the proposed techniques.

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Critical Point Constraint

Many optimization problems may be posed as follows: minimize $f(X)$ subject to the constraints $g_i(X) \geq 0$, $i=1,2,\ldots,n_g$, where X is a design vector. For many optimum design problems, constraints have to be satisfied for an entire range of certain parameters. In many cases, the design problem is characterized by constraint functions that must be enforced for an entire time interval,

$$g_i(X,t) \geq 0, \quad i=1,2,\ldots,n_g \qquad t_0 \leq t \leq t_f \tag{1a}$$

For example, stress constraints may be written as

$$g_i(X,t) = 1 - \sigma_i(t)/\sigma_a \tag{1b}$$

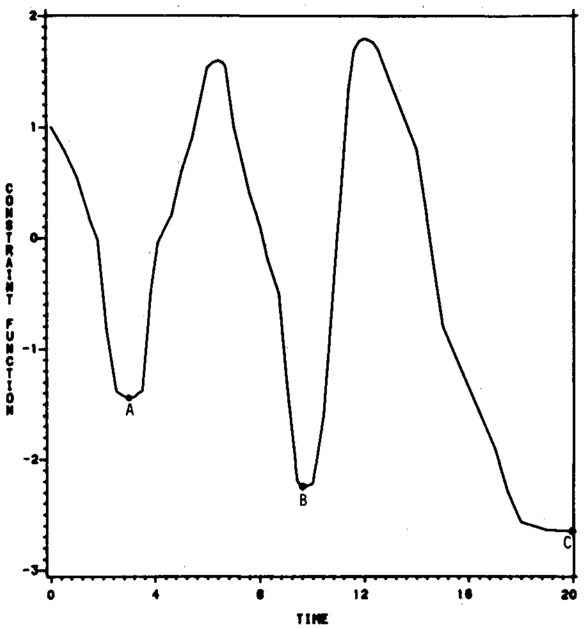

Fig. 1 Typical variation of constraint with time.

where $\sigma_i(t)$ is a stress component and σ_a the corresponding allowable stress.

One common way of enforcing a time-dependent constraint is to use a set of closely spaced time points t_j ($j=1,2,\ldots,n_t$) such that serious constraint violations at intermediate time points are unlikely. With this discrete time approach, the constraints are replaced by

$$g_{ij}(X) = g_i(X,t_j) \geq 0, \quad i=1,2,\ldots,n_g \qquad j=1,2,\ldots,n_t \tag{2}$$

In many practical applications, the product $n_g n_t$ is a very large number and carrying out the design process may become prohibitively expensive. The critical time point approach[8,9] replaces the monitoring of a constraint at all n_t time points by monitoring it only at its most critical points. This concept is explained by Fig. 1, which shows schematically the variation of a constraint function with time. The times of the local minima of the constraint function (A-C in Fig. 1) are herein called the critical time points. Instead of monitoring the constraint function at all times, it is monitored only at these times. However, the critical time points drift as the design changes and the constraint must be periodically calculated at many time points to accurately locate the critical time points.

The critical time points concept is useful because the drift of the critical time points may be neglected for portions of the design process. For these portions of the design process, the constraint function has to be evaluated for only a relatively small number of time points. In practice, critical time points are calculated periodically and are frozen between updatings. However, the concept remains useful, even if critical points require frequent updating, because of the large savings in the derivative calculation.

When the constraint function has more than one local peak as in Fig. 1, each critical point must be assigned a separate constraint g_{ci}. Otherwise, critical point constraints may have discontinuous derivatives when the global peak is switched from one local peak to another, even if g_i is a continuously differentiable function.

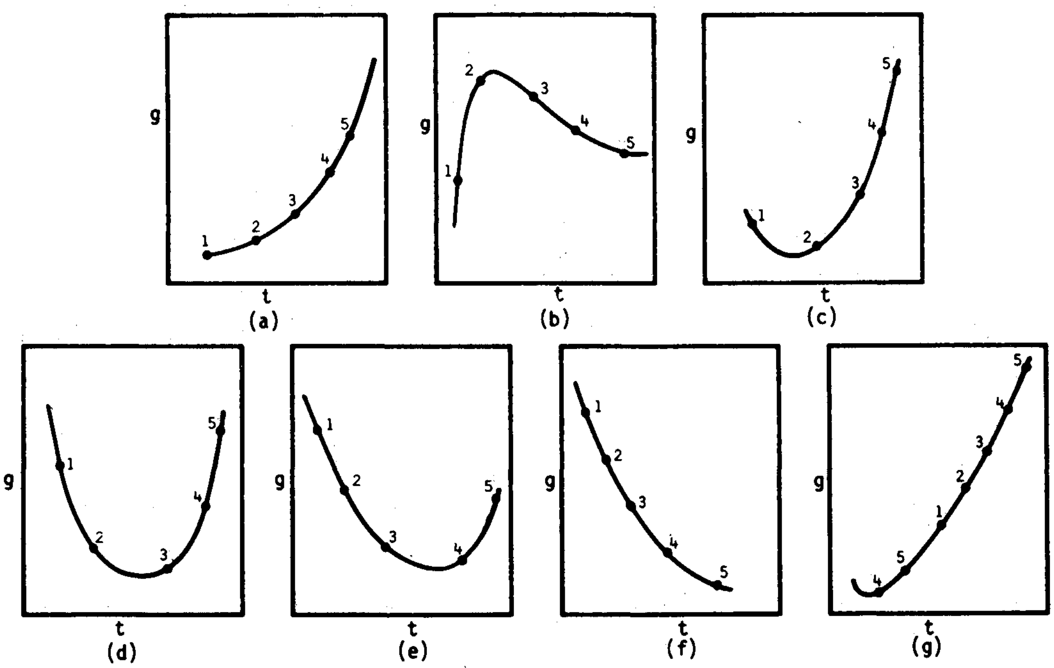

Fig. 2 Various possibilities that might arise in the constraint function.

Efficient Evaluation of Critical Points

Adaptive Search Technique

The adaptive search technique has the objective of finding all of the local minima of the constraint function.

We assume that the parametric constraint function data may be calculated only at uniformly spaced time points with an interval of α. We also assume that the derivatives of the constraint functions with respect to the variable t are not available. The adaptive search method first locates the approximate position of a critical point and then employs quadratic interpolation or fit to refine the approximation.

The following adaptive search algorithm is based on the premise that only critical points where $g(t)$ is below a cutoff value g_0 are of interest. Therefore, when $g(t)$ is larger than g_0, large steps can be taken, while when $g(t)$ is below g_0, the step size is reduced. The minimum and maximum step sizes are assumed to be given as $n_{\min}\alpha$ and $n_{\max}\alpha$, respectively. In the following outline of the search technique, t is always the latest position at which $g(t)$ is evaluated.

Step 1: Compute the constraint $g(t)$, at $t=t_0$.

a) If $g(t_0)>g_0$, set $n=n_{\min}$ and proceed to step 2.

b) If $g(t_0)<g_0$, calculate the constraint at $t=t_0+\alpha$ and, if $g(t)>g(t_0)$, then $g(t_0)$ is a minimum, so go to step 5. Otherwise, calculate $g(t)$ at $t=t_0+2\alpha$, $t_0+3\alpha,\ldots$ until $g(t)>g(t-\alpha)$. Then use $g(t-2\alpha)$, $g(t-\alpha)$, and $g(t)$ to identify the position of the minimum by quadratic interpolation. Go to step 5.

Step 2: Set $t=t+n\alpha$. If t is larger than t_f, then stop. Otherwise, compute $g(t)$.

Step 3: If $g(t)>g_0$, go to step 2 with $n=\min(n+1, n_{\max})$. Otherwise, go to step 4.

Step 4: Compute the constraint at p points on each side of the present time point and from the available $2p+1$ values find the lowest. It can be either one of the end points or an interior point. The choice of the parameter p depends on the problem and on $n_{\max}$. The following description is given for p equal to 2.

a) If the first point is the lowest (i.e., Figs. 2a and 2b), continue the search backwards with a step size of α, until $g(t-2\alpha)>g(t-\alpha)$ and $g(t)>g(t-\alpha)$. Estimate the critical time by quadratic interpolation.

b) If an interior point is the lowest (i.e., Figs. 2c-2e), construct the quadratic interpolant by taking one data value on each side of the minimum and estimate the critical time.

c) If the last point is the lowest (Fig. 2f), continue the search with a step size α until $g(t-2\alpha)>g(t-\alpha)$ and $g(t)>g(t-\alpha)$ (estimate the critical time by quadratic interpolation) or t reaches t_f [$g(t_f)$ is a minimum].

Step 5: Set $n=n_{\min}$.

Step 6: Set $t=t+n\alpha$. If t is larger than t_f, then stop.

Step 7: Evaluate $g(t)$. If $g(t)>g_0$, go to step 3. If $g(t)\geq g(t-n\alpha)$ (Fig. 2g), go to step 6. Otherwise, set $t=t-n\alpha$ and go to step 4c.

Besides three-point quadratic interpolation, a quadritic least squares fit based on five points was also employed to calculate the local minima. The least squares fit is less sensitive to slight errors in the data than the direct interpolation. Furthermore, an approximation based on five points gives a better estimation of the peak values than the interpolation based on three points.

Spline Approximation

Interpolation of data by polynomials of high degree is often unsatisfactory because they may exhibit wild oscillations. An alternative that provides a smoother approximation is based on splines. A "spline" is a function consisting of polynomial pieces on subintervals, joined together with certain smoothness conditions. Locally, the spline interpolant is always a low-degree polynomial regardless of the number of interpolation points and gives a high-quality approximation.

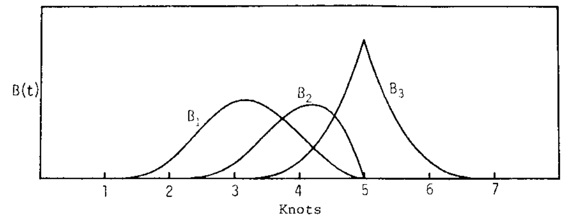

Fig. 3 Three cubic B-splines for the knot sequence (1, 2, 3, 5, 5, 5, 7).

Using splines, one has the choice of either interpolating or approximating the data. For a smooth and precisely known function, construction of an interpolant is effective and cheap, provided one has a reasonable way of choosing appropriate interpolation points. On the other hand, least squares approximation is called for when it is believed that the given data contain a slowly varying component (the true underlying function) and a comparatively fast-varying, small-amplitude component (the noise in the data). Least squares approximation is a global filter, with much more attractive properties than the simple local filters of signal processing. For noisy data, the degrees of freedom (the dimension of the spline vector space from which the approximation is chosen) for the spline are much less than the number of data points. There should be enough degrees of freedom to approximate the underlying slowly varying function well, but not enough to reproduce also the high-frequency noise. To this end, splines are very effective as an approximating family and discrete least squares spline approximation is very suitable for the recovery of a smooth function from noisy information.

In this work, least squares spline approximation is used because of its efficiency is representing noisy data. Splines of second degree are employed here, since the calculation of critical constraint values from quadratic equations is simple. A formal definition of "spline" is given below.

A knot sequence is a monotonically increasing sequence of numbers (e.g., 0, 1, 1, 2, 2, 3, 3, 3...). Breakpoints are the distinct values (strictly increasing, of course) among the knots (e.g., 0, 1, 2, 3...). Knot multiplicities at the breakpoints translate into smoothness conditions for the spline approximation. The spline approximation used here is computed with the aid of local basis spline functions B_i^k (the polynomial pieces have degree $\leq k-1$). For $i=1,\ldots,n$, the ith B-spline B_i^k of order k for the knot sequence $t=(t_1, t_2,\ldots, t_{n+k})$ is defined by the recurrence relation below (n is the dimension of the vector space generated by the B_i^k). The B-splines of order one for the knot sequence t are defined by

$$B_i^1(t) = \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

and the higher order B-splines are defined recursively by

$$B_i^k(t) = \left(\frac{t-t_i}{t_{i+k-1}-t_i}\right)B_i^{k-1}(t) + \left(\frac{t_{i+k}-t}{t_{i+k}-t_{i+1}}\right)B_i^{k-1}(t) \tag{4}$$

where $i=1,2,\ldots,n$.

Figure 3 shows the three cubic B-splines for the knot sequence (1, 2, 3, 5, 5, 5, 7) to demonstrate that the B-splines are piecewise polynomials of order four with support only over $k+1=5$ knots.

Formally, a "spline" is defined as a linear combination of B-splines. By the Schoenberg-Whitney theorem,[10] any piecewise polynomial can be represented as a linear combination of B-splines, with an appropriate choice of order k and knot sequence t. Hence, a spline can be alternatively described as a piecewise polynomial. The B-spline basis $\{B_i\}_{i=1}^{n}$ is numerically very stable and inexpensive to compute.

The algorithm used for least squares spline approximation is given in Ref. 11 and further details can be found in Ref. 12. In this study, the constraint function is approximated as a continuous function at the breakpoints, but the approximation does not have a continuous derivative.

The effectiveness of the least squares splines for noisy data is demonstrated through the following simple response function:

$$h(t) = \sin(t)\sin(2t) + 0.05\sin(30t) \qquad 0 \le t \le 5\pi \tag{5}$$

The first term in Eq. (5) represents a slowly varying smooth function and the second term represents fast varying noise. The function $h(t)$ is shown in Fig. 4 over the given time interval. A least squares spline approximation constructed using 50 piecewise polynomials is shown in Fig. 5. It is clear that the least squares spline approximation removed the fast varying noise and gave a good representation for the true underlying function.

The adaptive search technique described in the previous section is a local approximation approach based on interpolation, whereas splines provide a smoothed approximation. Once the spline function is constructed, the critical time points can be easily calculated using the derivative information at the spline breakpoints.

Superpeak Concept

The critical point approach is based on monitoring the constraint only at local minima or peaks. Once the peaks are identified, this approach can be carried one step further by grouping several closely spaced local peaks to identify the most critical one, called herein a "superpeak." The procedure for identifying superpeaks starts by locating the first peak with magnitude exceeding the cutoff value at time t_p. The highest peak in the time interval $(t_p, t_p + \Delta t_p)$ is taken as the first superpeak. The procedure is repeated starting at $t_p + \Delta t_p$ to locate the second superpeak.

The danger of monitoring only superpeaks is that, as the design is changed, the superpeak may switch from one local peak to another. This may result in a discontinuous derivative of the constraint with respect to the design variables. The switching phenomenon can also become a problem when derivative-based constraint approximations are employed. These constraint approximations replace the full analysis parts of the design process. Approximations based on the superpeak constraints do not account for the superpeak switching. Therefore, the errors associated with superpeak switching are compared here to the truncation errors in constraint function approximations used in structural optimization, such as the linear and reciprocal approximations. The simplest form of derivative based function approximation is the linear approximation

$$g_D(X) = g(X_0) + \sum_{i=1}^{\ell} (x_i - x_{0i}) \frac{\partial g}{\partial x_i}(X_0) \tag{6}$$

where X_0 is a nominal design vector and ℓ the number of design variables.

An alternate approximation, called the reciprocal expansion, is a first-order Taylor series in the reciprocals of the design variables. The reciprocal approximation is usually more accurate than the linear approximation for stress constraints in structures,[13]

$$g_R(X) = g(X_0) + \sum_{i=1}^{\ell} (x_i - x_{0i}) \frac{x_{0i}}{x_i} \cdot \frac{\partial g}{\partial x_i}(X_0) \tag{7}$$

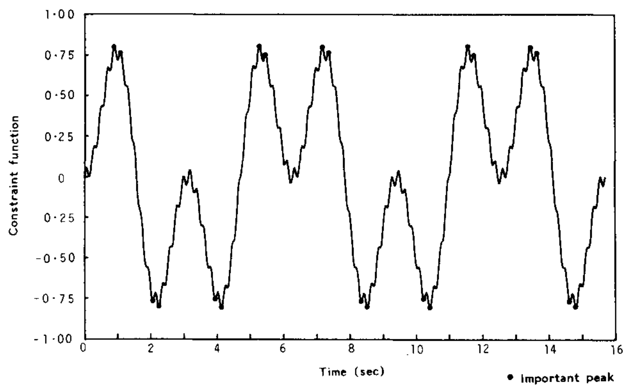

Fig. 4 Constraint behavior of a noisy function.

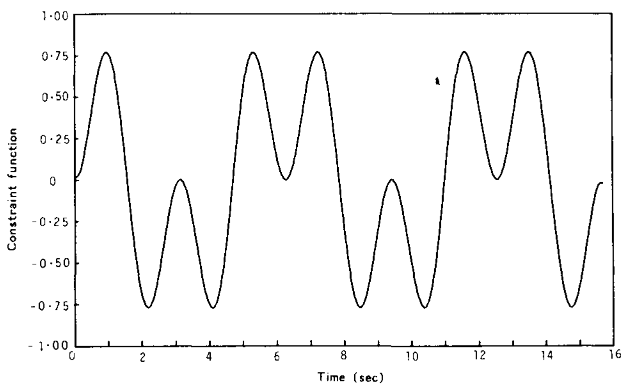

Fig. 5 Least squares spline approximation for noisy constraint function.

Results and Discussion

Two example problems are employed to demonstrate the adaptive search and the least squares spline approximation techniques. The first example is a simple test problem in which the exact results are known for the peak values and the second example is a 25 bar space truss structure. For the adaptive search technique, a quadratic interpolation based on three points and a least squares quadratic fit based on five points are used. For the least squares spline approximation, functions are approximated by quadratic splines continuous at breakpoints, but having discontinuous first derivative.

The accuracy of the calculated peaks is very important for the accuracy of the derivative calculation of the constraints

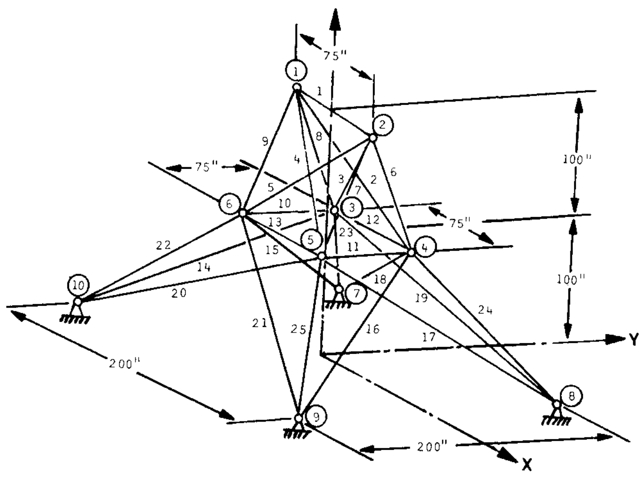

Fig. 6 Twenty-five bar space truss.

with respect to the design variables. These constraint derivatives play an important role in gradient-based optimization methods. In the following description, error percentages were calculated based on the absolute difference in peak values compared with the maximum amplitude of the response function. The peaks calculated with the adaptive search technique were allowed to have a maximum error of 1%. In the case of splines, root mean square (rms) errors were used for evaluating the quality of approximation. Errors in the rms smaller than the level of noise in the data were considered acceptable. Besides the rms error, the peaks were allowed an additional 1% error. The computational efficiency of the various techniques is assessed by comparing them to a "brute force" approach of calculating the stresses at equidistanced points and finding the peaks without resorting to approximation or interpolation.

The adaptive search technique assumes that the data are exact, whereas the least squares spline approximation assumes the data are noisy. For demonstration purposes, both problems were solved by first assuming that the data are exact and then noisy. The errors associated with the adaptive search and the least squares spline approach cannot be compared to each other because of the different assumptions on the accuracy of the data.

Analytical Example

The response function considered is given in Eq. (5). The local peaks greater (in absolute value) than 0.25 were considered as critical peaks. The peaks obtained with $\alpha = 0.0157$ s step size, combined with the quadratic interpolation, were taken as exact results for comparison. The "brute force" method of finding peaks required 350 function evaluations with a step size of 0.0449 s to achieve 1% accuracy. Forty peaks were identified during the 5π s time period. The number of 350 function evaluations was used for measuring the computational savings obtained by the adaptive search and splines techniques. Of the peaks shown in Fig. 4, some are obviously minor kinks and others are below the cutoff value of 0.25. A local peak was considered to be a mere kink if a local maximum was followed immediately by a local minimum differing by less than 3% of the response function maximum value. The function considered (Fig. 4) has several small kinks and, out of the 40 peaks, about 20 are considered to be important; these peaks are marked in Fig. 4. Only these important peaks were considered in evaluating the adaptive search technique.

First, the adaptive search technique with quadratic interpolation based on three points was employed. $n_{\min}$ and $n_{\max}$ were taken as 3 and 8, respectively. It required 216 function evaluations with $\alpha = 0.0449$ s to find the 28 peaks that included all of the important ones, which was a 38% savings in the number of function evaluations. The maximum error was about 0.5%. Increasing α further resulted in missing important peaks, so the results obtained with $\alpha = 0.0449$ s are used for comparison, even though the error percentages are smaller than 1%. Next, a quadratic fit based on five points was used for the adaptive search technique. It required 228 function evaluations with $\alpha = 0.0394$ s to find the 27 peaks that included all of the important ones. A savings of about 35% was realized in the number of function evaluations and the maximum error in the peaks was about 1%. Error comparisons for quadratic interpolation and quadratic fit are given in Table 1.

Next, the least squares spline approximation was employed using 50 polynomial pieces with equally spaced breakpoints. The exact magnitude of the peak values is known to be ± 0.7703 for the smooth function part corresponding to the first term in Eq. (5) and there are exactly 10 peaks. The rms error acceptable for the spline approximation was 5% (i.e., the known noise in the given data). In order to find 10 peaks within 1% accuracy and also to satisfy the rms error restriction, 150 function evaluations were required with a step size

Table 1 Comparison of critical constraint values accuracy using the adaptive search technique for analytical example

Exact solution $\alpha = 0.0157$ s		Quadratic interpolation $\alpha = 0.0449$ s		Percentage difference	Quadratic fit $\alpha = 0.0394$ s		Percentage difference
Time, s	$h(t)$	Time, s	$h(t)$		Time, s	$h(t)$	
0.5060	0.4363	0.5127	0.4400	0.46[a]	—	—	—
0.7049	0.6769	0.7150	0.6738	0.38	0.7317	0.6787	0.22
0.8963	0.8110[b]	0.8980	0.8110	0.00	0.9033	0.8038	0.89
1.0850	0.7755[b]	1.0801	0.7742	0.16	1.0774	0.7718	0.46
2.0566	−0.7755[b]	2.0615	−0.7742	0.16	2.0642	−0.7718	0.46
2.2453	−0.8110[b]	2.2436	−0.8110	0.00	2.2383	−0.8038	0.89
3.7483	−0.5641	—	—	—	3.7685	−0.5715	0.91
3.9426	−0.7623[b]	3.9478	−0.7612	0.12	3.9507	−0.7589	0.42
4.1326	−0.8165[b]	4.1311	−0.8163	0.02	4.1269	−0.8085	0.99[a]
5.1055	0.6886[b]	5.1166	0.6863	0.28	5.1320	0.6922	0.44
5.2922	0.8165[b]	5.2937	0.8163	0.02	5.2978	0.8085	0.99[a]
5.4822	0.7623[b]	5.4770	0.7612	0.14	5.4740	0.7589	0.42
6.7892	0.4363	6.7959	0.4400	0.46[a]	—	—	—
6.9880	0.6769	6.9982	0.6738	0.38	7.0149	0.6787	0.22
7.1794	0.8110[b]	7.1812	0.8110	0.00	7.1865	0.8038	0.89
7.3682	0.7755[b]	7.3633	0.7742	0.16	7.3606	0.7718	0.46
8.3398	−0.7755[b]	8.3447	−0.7742	0.16	8.3474	−0.7718	0.46
8.5285	−0.8110[b]	8.5268	−0.8110	0.00	8.5215	−0.8038	0.89
10.0314	−0.5641	—	—	—	10.0517	−0.5715	0.91
10.2257	−0.7623[b]	10.2310	−0.7612	0.14	10.2339	−0.7589	0.42
10.4157	−0.8165[b]	10.4143	−0.8162	0.02	10.4101	−0.8085	0.99[a]
11.3887	0.6886	11.3998	0.6863	0.28	11.4152	0.6922	0.44
11.5754	0.8165[b]	11.5768	0.8163	0.02	11.5810	0.8085	0.99[a]
11.7654	0.7623[b]	11.7601	0.7612	0.14	11.7572	0.7589	0.42
13.0723	0.4363	13.0791	0.4400	0.46[a]	—	—	—
13.2712	0.6769	13.2814	0.6738	0.38	13.2981	0.6787	0.22
13.4626	0.8110[b]	13.4644	0.8110	0.00	13.4697	0.8038	0.89
13.6513	0.7755[b]	13.6464	0.7742	0.16	13.6437	0.7718	0.46
14.6230	−0.7755[b]	14.6279	−0.7742	0.16	14.6306	−0.7718	0.46
14.8117	−0.8110[b]	14.8100	−0.8110	0.00	14.8047	−0.8038	0.89

[a]Maximum difference. [b]Important peak.

α of 0.1047 s. The peaks obtained using spline approximation had the value of ± 0.7698 or 0.07% off the exact value with 57% savings in the number of function evaluations. This example demonstrates that the spline approximation is successful in removing noise from the data and giving a good representation for the true underlying slowly varying function.

Twenty-Five Bar Space Truss

A 25 bar space truss structure (Fig. 6) subject to an earthquake loading (Fig. 7) is the next example. The truss is made from aluminum and the cross-sectional areas of all members are 5 cm^2. The structural analysis procedure is described in Ref. 11 and the stress response in each element was considered as a parametric time-dependent constraint. The stress response in a typical element is shown in Fig. 8 for a time period of 10.0 s. The goal of the procedure was to find all stress peaks greater (in absolute value) than 6.205 MPa (corresponding to g_0 in the analysis).

The peaks obtained using equispaced data points with $\alpha = 0.01$ s, combined with quadratic interpolation based on three points, were taken as exact for comparison, since reducing α further changed peak values very little. For element 24 (a typical element), 29 local peaks were identified. To achieve 1% error using equispaced time points ("brute force" approach), the stresses have to be evaluated at every 0.0133 s for a total of 750 stress evaluations.

Adaptive Search Technique

First, the adaptive technique was applied to identify the local peaks. The minimum and maximum step sizes, $n_{\min}$ and $n_{\max}$ were taken as 3 and 5, respectively. Peaks were calculated from a quadratic interpolation based on three points. For element 24, 29 peaks were identified using a step size of 0.02 s with 252 stress evaluations, a savings of 66% in the number of stress evaluations. The process was repeated using the same α, $n_{\min}$, and $n_{\max}$ for all elements to achieve more than 75% savings for the complete 25 bar truss structure. The peaks obtained using the adapative search technique have a maximum error of about 1% when compared with the results obtained using data points every 0.01 s with quadratic interpolation (Table 2).

Next, the three-point quadratic interpolation was replaced by a quadratic least squares fit based on five data points. Using a step size of 0.0118 s, 29 local peaks were identified with 406 stress evaluations, a savings of about 46% for element 24. For the entire structure, the corresponding saving in the number of stress evaluations was 63%. The results of the five-point quadratic fit are compared with the previous three-point quadratic interpolation in Table 2. The total number of stress evaluations required for the complete 25 bar space truss structure using the quadratic interpolation and quadratic fit techniques is given in Table 3.

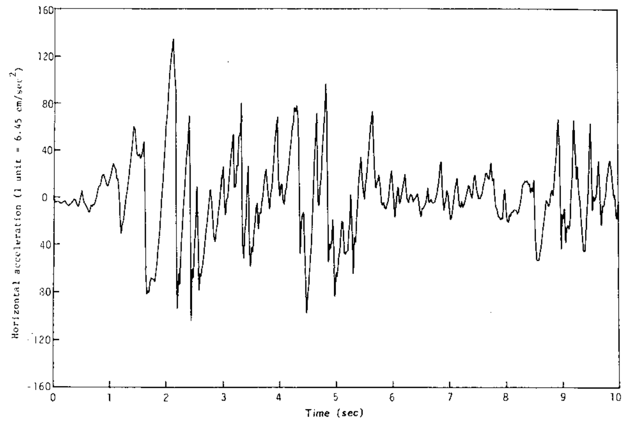

Fig. 7 Earthquake data.

Table 2 Accuracy of critical stresses using adaptive search technique for element 24 of 25 bar truss

Exact solution $\alpha=0.01$ s		Quadratic interpolation $\alpha=0.02$ s		Percentage difference	Quadratic fit $\alpha=0.0118$ s		Percentage difference
Time, s	Stress, MPa	Time, s	Stress, MPa		Time, s	Stress, MPa	
1.3977	10.117	1.3993	10.111	0.01	1.4000	10.095	0.08
1.7422	−20.698	1.7421	−20.765	0.25	1.7419	−20.737	0.14
2.0952	22.997	2.0963	22.978	0.07	2.0965	22.952	0.16
2.2596	−24.435	2.2603	−24.434	0.00	2.2606	−24.374	0.22
2.3943	24.051	2.3945	24.026	0.09	2.3948	24.014	0.14
2.5078	−28.852	2.5080	−28.951	0.36	2.5088	−28.684	0.61
2.6214	10.199	2.6210	10.192	0.03	2.6207	10.055	0.52
2.7206	−20.580	2.7207	−20.581	0.00	2.7209	−20.439	0.51
2.8301	13.588	2.8302	13.539	0.18	2.8302	13.383	0.74
2.9390	−16.337	2.9389	−16.338	0.00	2.9388	−16.265	0.26
3.0434	14.440	3.0443	14.480	0.15	3.0452	14.221	0.79
3.1507	−7.326	3.1507	−7.326	0.00	3.1501	−7.222	0.38
3.2614	11.838	3.2618	11.840	0.01	3.2620	11.794	0.16
3.4010	−10.704	3.4021	−10.721	0.06	3.4025	−10.695	0.03
3.9828	13.490	3.9824	13.472	0.07	3.9830	13.331	0.58
4.1069	−8.383	4.1065	−8.336	0.17	4.1065	−8.260	0.45
4.2374	14.642	4.2383	14.635	0.03	4.2386	14.605	0.14
4.5075	−14.235	4.5074	−14.227	0.03	4.5083	−14.129	0.38
4.6778	15.559	4.6776	15.563	0.02	4.6774	15.322	0.86
4.8620	16.543	4.8623	16.548	0.02	4.8623	16.237	1.11
4.9974	−22.793	4.9968	−22.813	0.07	4.9965	−22.708	0.31
5.2221	−14.895	5.2222	−14.895	0.00	5.2221	−14.771	0.45
5.6846	11.843	5.6845	11.837	0.02	5.6845	11.751	0.34
8.6073	−11.101	8.6079	−11.136	0.13	8.6086	−11.081	0.07
8.9687	13.515	8.9679	13.232	1.03[a]	8.9664	13.197	1.15[a]
9.0958	−13.972	9.0954	−13.981	0.04	9.0947	−13.817	0.56
9.2385	13.082	9.2375	13.097	0.06	9.2367	12.916	0.60
9.3921	−10.389	9.3921	−10.397	0.03	9.3918	−10.355	0.12
9.5395	12.196	9.5393	12.198	0.01	9.5391	12.013	0.67

[a]Maximum difference.

Least Squares Spline Approximation

The use of least squares splines is called for when the data are assumed to be noisy. For the purpose of demonstration, it was assumed that the noise amplitude is 3% of the maximum stress amplitude. Besides the rms error, the peaks were allowed to have 1% additonal error and thus the total error allowed in the peaks was 4%. Spline techniques require the selection of the location of the breakpoints. For this purpose, stresses were calculated at a small number of points; critical regions were identified based on the magnitude of stress values. About two-thirds of the breakpoints were uniformly spaced in critical regions to achieve the maximum possible accuracy in peak stress values and the remaining breakpoints were uniformly spaced in other regions.

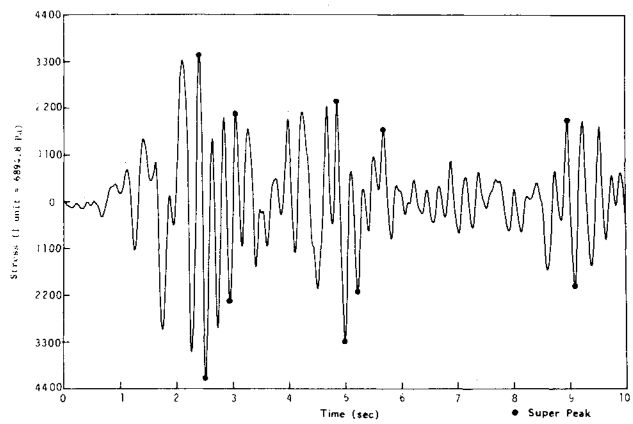

Fig. 8 Stress response for element 24 of 25 bar truss.

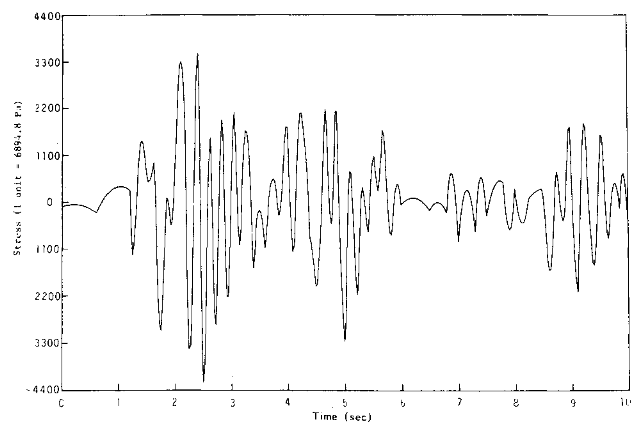

Fig. 9 Least squares spline approximation for stress response in element 24.

Table 3 Number of function evaluations required for 25 bar space truss structure

Members	Equispaced stress calculation	Quadratic interpolation $\alpha = 0.02$ s	Quadratic fit $\alpha = 0.0118$ s
1	750	101	171
2-5	750	182	314
6-9	750	168	290
10-11	750	118	201
12-13	750	101	171
14-17	750	164	257
18-21	750	137	249
22-25	750	252	406
Total no. of stress calculations	18,750	4151	6979

The restrictions on rms errors and peak errors were satisfied with 400 stress evaluations in each element using 90 piecewise polynomials. The approximate stress in element 24 obtained using least squares parabolic splines is shown in Fig. 9. Identified for element 24 were 29 peaks with 400 stress evaluations, a savings of 46%. The stress peak values have a maximum difference of about 4%. A savings of over 46% was realized in the number of stress evaluations for the complete structure. The spline approximation is more costly than the adaptive search technique. However, unlike the adaptive search technique, it has the potential of removing part of the noise in the data.

Superpeaks

Superpeaks were calculated by grouping local peaks occurring in each interval of 1.0 s duration. Positive and negative

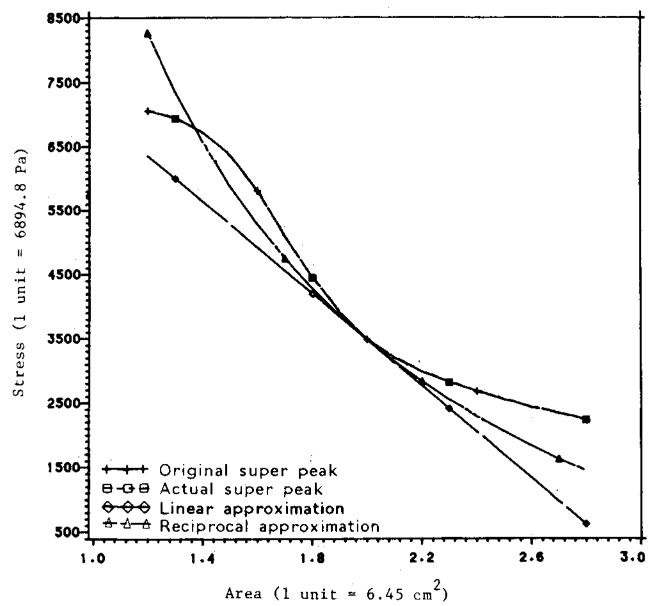

Fig. 10 Study of superpeak switching at 2.3945 s for element 24 during design modification process.

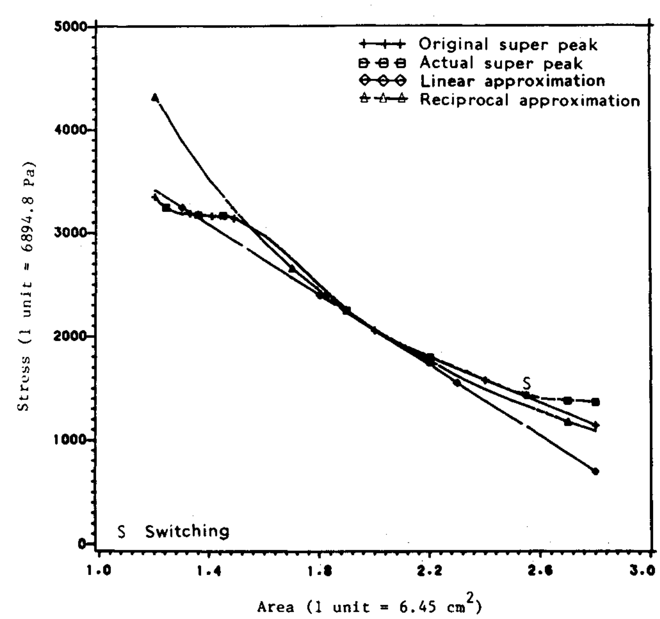

Fig. 11 Study of superpeak switching at 4.862 s for element 24 during design modification process.

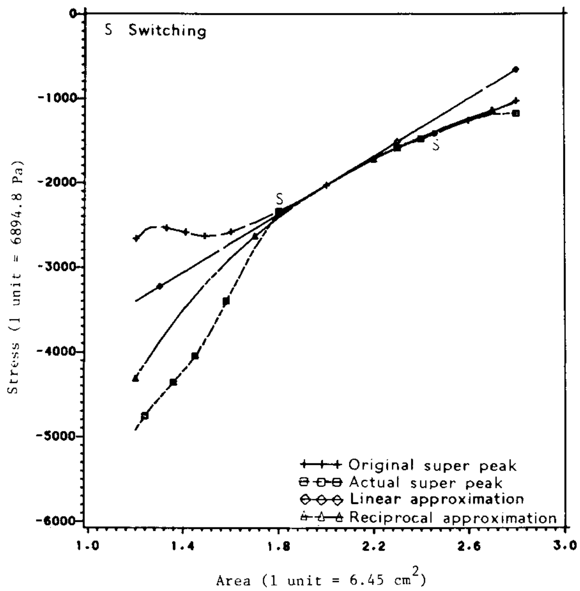

Fig. 12 Study of superpeak switching at 9.096 s for element 24 during design modification process.

peaks were grouped separately. For element 24, 10 of the 29 local peaks were identified as superpeaks. The reliability of the superpeak concept was studied by scaling all the cross-sectional areas by ±40% about the nominal design and monitoring superpeak switching. The error associated with superpeak switching was compared to errors due to other types of constraint approximations used in structural optimization. Stress derivatives were computed using a finite difference scheme for constructing stress approximations using Eqs. (6) and (7). The error comparison is shown in Figs. 10-12 by constructing approximations at three different critical time points for element 24 for a nominal design in which the areas of all elements was 5.0 cm^2. The four curves in Fig. 10 show the stress value monitored at the original superpeak (i.e., the maximum of the stress in a particular time zone), the maximum of the actual superpeak value in that particular time zone, and the linear and reciprocal approximations to the original superpeak as the design variables were changed. Figure 10 shows no switching of the superpeak. Superpeak switching was observed only for design change larger than ±25% in Fig. 11. In Fig. 12, superpeak switching took place for even small design changes. The location of superpeak switching is marked by the letter S in Figs. 11 and 12. It is seen from Figs. 10-12 that the superpeak approximation does not result in excessive errors for moderate changes in the structural properties and, also, the errors involved are comparable to errors due to commonly used approximations.

Conclusions

Three algorithms were developed for identifying critical times in the transient response of dynamically loaded structure. The first technique is an adaptive search technique in which the magnitude of the constraint at a particular time point determines the time step for the next constraint evaluation. The technique is well suited for a slowly varying, exactly known transient response. The least squares spline approximation is suggested for a noisy response to obtain the true underlying function. Two examples were used to show that the adaptive search and the least squares spline approximation techniques can be used for efficiently finding critical times with good accuracy.

A concept of grouping peaks and analyzing only "superpeaks" within clusters of local peaks was investigated and it is very useful for a fast varying structural response with many critical points. It reduces the computational effort involved in the optimization due to the reduced number of design constraints. The error incurred due to superpeak switching was shown to be comparable to errors due to other constraint approximations commonly used in optimization.

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